SOBOLEV FAZOSIDA KOSHI TIPIDAGI SINGULYAR INTEGRALLARNI OPTIMAL KVADRATUR FORMULALAR YORDAMIDA TAQRIBIY HISOBLASH

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Annotatsiya. Ushbu ishda Sobolev fazosida Koshi yadroli singulyar integrallarni taqribiy hisoblash uchun optimal kvadratur formulalarni qurish masalasi tadqiq qilingan . Sard ma'nosidagi optimal kvadratur formulaning koeffitsiyentlari teng qadamli tugunlar uchun oshkor ko'rinishda topilgan . Olingan formulaning yuqori aniqligi va samaradorligi amaliy test integrallari yordamida diskret o'ramalar usullari bilan taqqoslanib, sonli tahlillar asosida to'liq asoslab berilgan.

Kalit so'zlar: Sobolev fazosi, Koshi tipidagi singulyar integral, optimal kvadratur formula, xatolik funksionali, diskret o'ramalar metodi, aniqlik tahlili.

Differensiallanuvchi funksiyalarning turli funksional fazolarida Koshi va Gilbert yadroli singulyar integrallarni taqribiy hisoblash uchun optimal kvadratur formulalarni qurish hisoblash matematikasining fundamental vazifalaridan biridir. Mazkur masala xarakteristik singulyar integral tenglamalarning taqribiy yechimlarini tahlil qilish bilan bevosita bog'liq .

Tadqiqotda quyidagi kvadratur formula:

$$\int_{-1}^{1} \frac{\varphi(x)}{x-t}dx \cong \sum_{\beta=0}^{N} C[\beta]\varphi(x_\beta) \eqno(1)$$

hamda unga mos keluvchi xatolik funksionali o'rganiladi:

$$l(x)=\frac{\epsilon_{[-1,1]}(x)}{x-t}-\sum_{\beta=0}^{N} C[\beta]\delta(x-x_\beta) \eqno(2)$$

bu yerda $-1<t<1$, $C[\beta]$ — noma'lum koeffitsiyentlar, $x_\beta \in [-1,1]$ — formulaning tugunlari, N — natural son, $\epsilon_{[-1,1]}(x)$ — $[-1,1]$ kesmaning xarakteristik funksiyasi, δ — Dirak delta-funksiyasi, $\varphi \in L_2^{(2)}(-1,1)$. $L_2^{(2)}(-1,1)$ Sobolev fazosi 2-tartibli umumlashgan hosilasining kvadrati integrallanuvchi funksiyalar to'plami bo'lib, unda norma quyidagicha kiritiladi:

$$\|\varphi|L_2^{(2)}(-1,1)\|=\left\{\int_{-1}^{1}\left(\varphi^{(2)}(x)\right)^2 dx\right\}^{\frac{1}{2}}$$

(2) xatolik funksionali $L_2^{(2)}(-1,1)$ fazoda aniqlangan bo'lib, (1) formulaning mavjud bo'lishi uchun quyidagi shartlarni qanoatlantirishi lozim ($N\geq 1$):

$$(l,x^\alpha)=0, \quad \alpha=0,1 \eqno(3)$$

Tugun nuqtalar fiksirlangan holda xatolik funksionali normasini minimallashtirish (Sard masalasi) orqali teng qadamli to'r uchun optimal koeffitsiyentlar aniqlandi :

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1-teorema. $L_2^{(2)}(-1,1)$ fazoda (1) optimal kvadratur formulaning koeffitsiyentlari teng qadamli tugunlarda quyidagi analitik shaklga ega bo'ladi:

$$\tilde{C}[0]=\frac{6}{h^3}\left[f_2(h)-7f_2(0)-p_1^- h+\frac{g_0}{12}(3h+3h^2+h^3)+\frac{g_1}{4}(2h+h^2)\right]+$$
$$+\frac{36(1-q_1)}{h^3(1+q_1)}\left[\sum_{\gamma=0}^{N} q_1^{\gamma} f_2(h\gamma)+M_1+q_1^N N_1\right]$$

[cite: 13, 14]

$$C[\beta]=\frac{6}{h^3}\left[f_2\big(h(\beta-1)\big)-8f_2(h\beta)+f_2\big(h(\beta-1)\big)\right]+$$
$$+\frac{36(1-q_1)}{h^3(1+q_1)}\left[\sum_{\gamma=0}^{N} q_1^{|\beta-\gamma|} f_2(h\gamma)+q_1^{\beta}M_1+q_1^{N-\beta}N_1\right],\quad \beta=1,2,\ldots,N-1$$

[cite: 14]

$$\tilde{C}[N]=\frac{6}{h^3}\left[f_2(2-h)-7f_2(2)+p_1^+ h+\frac{g_0}{12}(3h+3h^2+h^3)-\frac{g_1}{4}(2h+h^2)\right]+$$
$$+\frac{36(1-q_1)}{h^3(1+q_1)}\left[\sum_{\gamma=0}^{N} q_1^{|N-\gamma|} f_2(h\gamma)+q_1^N M_1+N_1\right]$$

[cite: 14]

(Eslatma: $x_\beta=h\beta-1$ tuguniga to'g'ri keladigan yagona t nuqtada C[β] koeffitsiyentlar nolga aylanadi[cite: 14]).

Taklif etilgan usulning aniqligini baholash maqsadida Maple paketida quyidagi model integral hisoblandi:

$$\int_{-1}^{1}\frac{2x^2-5x+10}{x-t}dx$$

[cite: 14]

Mazkur integral Diskret o'ramalar metodi (DOM), uning modifikatsiyasi (DOMM) hamda yangi qurilgan Optimal kvadratur formula (OKF) yordamida hisoblandi:

- h=0.1 qadamda (N=20) DOM xatoligi 0.25−3.39, DOMM xatoligi $10^{-3}-10^{-2}$ tartibida bo'lganda, OKF xatoligi $10^{-5}-10^{-4}$ tartibni tashkil etdi[cite: 14].
- h=0.02 qadamda (N=100) esa OKF ning xatoligi $1.5\cdot 10^{-8}-1.5\cdot 10^{-5}$ gacha kamayib, DOM ($10^{-1}-3.58$) va DOMM ($10^{-5}-10^{-2}$) usullaridan ancha ustun ekanini ko'rsatdi[cite: 15].

Xulosa. Qiyosiy sonli natijalar va xatoliklar tahlili Koshi tipidagi singulyar integrallarni hisoblashda Sobolev fazosida qurilgan optimal kvadratur formulalar yuqori aniqlik va tez yaqinlashishni to'liq ta'minlashini isbotlaydi .

Adabiyotlar

1. Shadimetov Kh.M., Hayotov A.R., Akhmedov D.M., Optimal quadrature formulas for Cauchy type singular integral in Sobolev space. Applied Mathematics and Computation, 2015, No.1, 302-314.[cite: 15]

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2. Shadimetov Kh.M., Nuraliev F.A., Toshboev O.N., Optimal quadrature formulas for the approximate calculation of the Riemann-Liouville fractional order integral. AIP Conf. Proc. 3004, 060041 (2024).[cite: 15]
3. Shadimetov Kh.M., Nuraliev F.A., Toshboev O.N., Optimal methods for approximate calculation of the fractional Riemann-Liouville integral in Sobolev space. AIP Conf. Proc. 3147, 030031 (2024).[cite: 15]

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