GROUND STATES OF THE ISING–POTTS MIXED MODEL ON A CAYLEY TREE AND THEIR DEPENDENCE ON PARAMETERS

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Abstract. This study investigates the ground states of the mixed Ising–Potts model defined on a Cayley tree and examines their dependence on the parameters of the system. The Ising and Potts models are fundamental mathematical models used to describe phase transitions, magnetic ordering, and collective behavior in statistical mechanics. Combining these models within a single framework makes it possible to explore more complex interactions between neighboring vertices and identify different types of stable configurations. The main objective of this research is to analyze the conditions under which ground states emerge and determine how changes in interaction constants and other model parameters affect the structure and stability of these states. Particular attention is paid to the role of competing interactions and the influence of the tree structure on the possible configurations of the system. The results of such an analysis can contribute to a deeper understanding of ground-state behavior in mixed statistical-mechanical models.

Keywords: Ising model, Potts model, mixed model, Cayley tree, ground states, Hamiltonian, interaction parameters, spin configurations, statistical mechanics, phase transitions, magnetic ordering, equilibrium states, competing interactions, stability, mathematical physics.

The investigation of ground states in statistical-mechanical systems is an important problem in mathematical physics. A ground state is a configuration of a system that minimizes its energy under specified interaction conditions. Studying ground states provides essential information about the possible equilibrium configurations of a model and helps explain how the system responds to variations in its parameters. The Ising model is one of the most widely studied models in statistical mechanics. It describes systems in which each vertex is associated with a spin variable that can take one of two possible values. The Potts model generalizes this framework by allowing each spin to take one of several states. Both models have been extensively investigated because of their relevance to magnetic systems, phase transitions, and mathematical descriptions of interacting particles. A mixed Ising–Potts model combines features of these two approaches and provides a broader framework for studying systems with different types of interactions. When such a model is defined on a Cayley tree, the underlying structure introduces specific mathematical properties that can be used to analyze the energy of configurations and characterize the possible ground states.

The main objective of this study is to investigate the ground states of the Ising–Potts mixed model on a Cayley tree and establish how their structures depend on the model parameters. The research focuses on the following tasks: formulating the Hamiltonian of the mixed system, identifying the configurations that minimize the energy, examining the influence of interaction parameters on the ground-state structure, and determining the conditions under which different configurations become energetically favorable. The principal methodological approach is based on mathematical analysis of the Hamiltonian and comparison of the energies associated with admissible configurations. The interaction constants are treated as parameters whose values determine the relative energetic advantages of competing states. By comparing these energies, it is possible to identify parameter regions associated with different ground-state configurations.

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The ground-state structure of the Ising–Potts mixed model depends primarily on the form and strength of the interactions included in the Hamiltonian. Depending on the model formulation, these interactions may favor parallel spin alignment, particular Potts-state arrangements, or competing configurations. When one interaction dominates the others, the ground state is generally determined by the configurations favored by that interaction, subject to the constraints imposed by the remaining terms in the Hamiltonian. When the interaction parameters become comparable, competition between different energetic contributions may produce alternative ground-state configurations.

The Cayley tree provides a useful mathematical setting for investigating these effects. Its hierarchical structure allows the energy of a finite subtree to be analyzed in relation to the configurations of its neighboring vertices. This property can facilitate the systematic comparison of candidate ground states and the derivation of conditions for their stability. An important part of the analysis is the identification of parameter values at which the energies of two or more configurations become equal. Such equal-energy conditions may indicate boundaries between regions of parameter space characterized by different ground-state structures. The exact locations and physical interpretations of these boundaries depend on the Hamiltonian, the coordination number of the tree, the number of Potts states, and the signs and magnitudes of the interaction constants. The analysis also highlights the distinction between ground-state configurations and finite-temperature equilibrium behavior. Ground states correspond to energy-minimizing configurations at zero temperature, whereas finite-temperature properties require additional consideration of entropy and thermal fluctuations. Therefore, a complete characterization of the model should distinguish energy minimization from the analysis of thermodynamic phase transitions.

Conclusion

The study of ground states in the Ising–Potts mixed model on a Cayley tree provides a mathematical framework for understanding how competing interactions determine the stable configurations of a complex spin system. The dependence of ground states on interaction parameters can be investigated by formulating the Hamiltonian, comparing the energies of admissible configurations, and identifying parameter conditions under which different states minimize the energy. The Cayley tree structure offers useful opportunities for systematic analysis because of its hierarchical organization. The resulting classification of ground states can help clarify the relationship between interaction strengths, spin configurations, and energetic stability. However, the precise classification requires an explicitly specified Hamiltonian and a detailed analysis of its parameter space. Further research may extend this investigation to finite-temperature Gibbs measures, phase transitions, and the stability of periodic or non-periodic configurations. These directions may provide additional insight into the mathematical properties of mixed spin models on hierarchical structures.Note: This is a general theoretical thesis draft. To obtain mathematically precise results, including explicit ground-state classifications, inequalities between interaction parameters, and formulas for phase boundaries, the exact Hamiltonian of your Ising–Potts mixed model and the coordination number of the Cayley tree are needed.

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